Optical Properties of Left-Handed Materials

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Abstract

Recently materials with the unusual property of having a simultaneously negative permeability and permittivity have been tested to verify predicted [1] optical properties. The most important of these properties is a left-handed wave vector orientation, meaning the direction of energy flow due to the wave is in the opposite direction of the wave vector. We have written a program to model the predicted behavior of these materials given the particulars of their construction, and have compared our predictions to some of these recent experiments. This comparison verified some predicted behaviors of these materials. We were also able to identify several factors contributing to the high absorption found in the specific materials considered.

INTRODUCTION

For a material to transmit electromagnetic waves, it must have a real index of refraction, and therefore its permittivity and permeability (ε and µ) must have the same sign. In all known cases in which this is true, the material has both positive ε and µ, but theoretically a material in which both ε and µ are both negative would also allow the propagation of electromagnetic waves. It has been predicted that such a material would exhibit several strange properties [1], including a left-handed wave vector orientation. Although metals have a negative ε, no known material exists in nature with a simultaneously negative ε and µ. Therefore it is necessary to fabricate such a material in order to verify its predicted behavior.

Because the properties of this kind of material have not been thoroughly experimentally explored, there exists an opportunity to verify and extend theoretical predictions, and also to find other non-obvious consequences of left-handedness.

Recently, certain structures have been shown to exhibit a negative permeability under some circumstances, thus allowing the construction of left-handed “meta-materials” (LHMs). These meta-materials are now being built with lattice constants on the order millimeters, but their smallness is not limited in theory. Experiments published in 2000 [2] and 2001 [5] have tentatively confirmed some theoretical predictions.

BACKGROUND

Through analyzing the case in which ε and µ are negative, several interesting properties become apparent. Maxwell’s equations state:

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{1}{c} \frac{\partial D}{\partial t}$$

In an electromagnetic wave \(E, D, B, H \propto e^{i(k \cdot r - \omega t)}\)

so

$$k \times E = \frac{\omega}{c} \mu H$$

$$k \times H = -\frac{\omega}{c} \varepsilon E$$

If ε and µ are positive, the typical result \(\hat{k} = \hat{E} \times \hat{H}\) follows (where the hat indicates the normalized vector). However, if ε and µ are negative then, \(-\hat{k} = \hat{E} \times \hat{H}\) forming a left-handed set of vectors. Thus the power flux, represented by the Poynting vector

$$S = \frac{c}{4\pi} E \times H'$$

is in the direction opposite to the phase velocity in an LHM.

This property leads to a variety of phenomena; some of which have been theoretically explored by Veselago [1]. Among Veselago’s results are the prediction of a reversed Doppler effect, and a reversal of Snell’s law and the direction of Cerenkov radiation. Veselago also briefly explores how the reversal of Snell’s law reverses the effects of convex and concave lenses, as well as causes the light from a point source on one
side of a LHM block to be focused at a point on the opposite side.

THEORY

The structure on which we have focused our attention is the split-ring Resonator (SRR). This device is constructed from two concentric conducting rings, where both rings have a small gap in them so current may not freely flow around either one independently. The SRR has been predicted to exhibit a negative $\mu$ for some frequencies determined by the specifics of its construction.

Pendry et al. [6] have derived the following expression for the effective permeability of a lattice of split ring resonators:

$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2/r}{\omega \mu_0} i - \frac{3\ell}{\pi \mu_0 \omega^2 C r^3}} \quad (1)$$

where $r$ is the inner radius of the resonator, is the transverse spacing (perpendicular to the wave vector) of the resonators, $a$ is the horizontal spacing (parallel to the wave vector), $r$ is the resistance per unit length of the rings, $C$ is the capacitance per unit length between the rings, and $w$ is the angular frequency of the incident wave. In this case, no current can flow around the structure, but transient currents in the two rings caused by a fluctuating magnetic field can be added to yield an effective current around the ring. The currents in the two split ring elements are related by the capacitance between the rings. This induced current in turn generates its own field, and the effective permeability can be found by taking the ratio of the driving magnetic field to the resulting magnetic field. We were able to re-derive Pendry’s result except for the factor of 3 in the last term in the denominator, but found that our simulations much more accurately modeled observed behavior by including the factor of 3. Theoretical curves produced by our program depending on this model therefore use Pendry’s equation.

For the capacitance in (1), Pendry uses the following:

$$C = \frac{\varepsilon_0}{\pi} \ln \left( \frac{2c}{d} \right) \quad (2)$$

where $c$ is the width of a ring, and $d$ is the separation between the rings. This contrasts sharply with the equation we derived:

$$C = \frac{2\pi \varepsilon_0}{\ln \left( \frac{r + c + d}{r + c} \right)} \quad (3)$$

where $\omega_\mu_\pi$ is the magnetic plasma frequency, defined by Shelby et al. as the resonant frequency of the SRR, and $\omega_\mu_0$ is the frequency of the low frequency edge of the observed stop band. This formula yields results similar to Pendry’s model (Figure 1). Both formulae predict an effective permeability which remains nearly constant ($\mu = 1$), except near the resonant frequency, where there is a single oscillation in which $\mu$ may drop below zero. This oscillation is accompanied by a spike in the imaginary part of $\mu$, which leads to a large absorption in this region. By reducing the resistivity of the materials from

![Reversal of Snell's Law](image)

**Figure 1:** Real and imaginary parts of the effective $\mu$ for Pendry’s sample SRR ($a = 0.01 \text{ m}$, $= 0.002 \text{ m}$, $r = 0.002 \text{ m}$, $C = 8.5 \times 10^{-12} \text{ F}$, $r = 200 \text{ W/m}$).
which the SRR is made, the imaginary spike can be made sharper, thus leading to a larger frequency band in which both \( \epsilon \) and \( \mu \) are negative and absorption is small.

It is important to note that a lattice of SRRs is not, in general, a LHM, even in the frequency regime in which the effective permeability of the lattice drops below zero. This is because the effective permittivity remains positive. In order to create a LHM, metal wires or rods can be set up next to each SRR in the lattice. This effectively endows the material with the permittivity of a metal (which is negative in general) while having no effect on its permeability.

\[
\mu_{\text{eff}} = 1 - \frac{\omega_n^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}
\]

The Program

We have written a program that calculates the transmission, reflection, and absorption coefficients of an object composed of periodic layers of materials for which the permittivities and permeabilities are known as functions of wavelength (using, for example, the formulas discussed above). The purpose of writing this program was twofold. First, by comparing the computer output with data from actual experiments it allows us to verify the accuracy of the theoretical models mentioned earlier. Second, by changing the input parameters of the program we are able to observe the effect of certain variables of the material's construction on transmission and absorption curves without resorting to experiment.

METHOD

The coefficients of transmission and reflection are calculated using a transfer matrix as described in Pedrotti & Pedrotti [3]. This method assigns a transfer matrix to each layer of material through which the wave travels which relates the \( E \) and \( H \) fields on each side of the material. The program calculates \( \epsilon \), \( \mu \), the index of refraction, and the wave impedance for a range of wavelengths, and uses these values to compute the transfer matrices for each layer at a given wavelength. By multiplying these matrices together in the order in which the light encounters the layers, a relation between the incident, transmitted, and reflected fields can be determined.
Capabilities

This program allows complex indices of refraction to account for absorptive effects. $e$ and $m$ may be specified to be any real or imaginary constant, or may be set equal to one of several special functions defined in the program. Any number of periodic double-layers may be specified. Coefficients for TE and TM modes are calculated separately, and averaged to yield the result for non-polarized light. The angle of incidence can also be specified, as may the range of wavelengths to be considered.

Testing

The program has been able to reproduce exactly all relevant examples given in Pedrotti & Pedrotti, as well as to reproduce the curve produced by another model created at UMass Lowell [4] of the reflectivity of a material made from layers of GaAs and AlGaAs. The TE and TM modes were successfully tested against cases given in Pedrotti & Pedrotti. Also, we were able to reproduce a curve given in two published articles [2, 5] reporting results of experiments measuring the transmission through a SRRs and a LHM.

RESULTS

Simulation of Shelby et al.’s experiment

An experiment to find the transmission through a row of SRRs was done by Shelby et al. [5]. Using the experimentally derived equations for $e$ and $m$ given in that paper our computer simulation (Figure 4) accurately reproduced the observed experimental data (dotted line in Figure 2). Indeed, the results of the computer simulation matches the experimental data more closely than the predictions given by Shelby (dotted line in Figure 3).

Shelby also conducted an experiment in which conducting wires were attached to the rings in order to cause a negative $e$, thus making a LHM at some frequencies. However, not enough information was given in their paper for us to be able to simulate this experiment.

Simulation of Smith et al.’s experiment

We attempted to use Pendry’s split-ring result to simulate the results of an experiment done by Smith et al. [2] in which the transmission through a lattice of SRRs was measured. Smith repeated this SRR experiment with conducting wires attached to the split-rings which cause the meta-material to respond to electric fields like a plasma, thus giving a negative effective permittivity for frequencies below the plasma frequency. Therefore, in the case in which the wires are included, the meta-material is an LHM for a range of frequencies below the plasma frequency of the wires. In either case, the resonant frequency of the meta-material is determined by the reso-
nant frequency of the SRRs. We first tried to predict this frequency using Pendry’s equation (1):

$$\nu_{res} = \frac{1}{2\pi} \sqrt{\frac{3\ell}{\pi^2 \mu \rho r^3 C}}$$

Using Pendry’s formula for capacitance (2), this frequency is predicted to be approximately 49 GHz; using our formula (3) the resonance is predicted at about 4.7 GHz, a difference of an order of magnitude. The observed resonance was about 4.8 GHz which is quite close to our prediction using our capacitance. A difference in the predicted and observed resonance frequency is expected since our capacitance was calculated using a crude model.

The transmission coefficient yielded by our computer simulation of the SRR experiment (solid line in Figure 6) seems to be reasonably close to that found experimentally by Smith (solid line in Figure 5). One difference is that the dip in the experimental result has a floor at around –35 dB. This floor, well above the noise floor of the detector, cannot be explained by our theoretical model, but may be due to the inability of the experimenter to produce a monochromatic wave. This type of inaccuracy would produce a shallowing of the trough and, possibly, the floor effect seen in the experimental data.

The transmission coefficient yielded by our simulation of the LHM experiment (solid line in Figure 7), though peaking at the correct frequency, was several orders of magnitude weaker than the observed transmission (dashed line in Figure 5). We were able to correct this problem by reducing the assumed resistivity of the SRR material by about 50. Since we did not know the resistivity of the material used in the first place, the correction is in fact not drastic. Note that, in any case, significant absorption effects are present, and even in the pass-band the transmission coefficient at best barely reaches 1%.

Simulations of SRRs, LHMs

We also calculated the transmission through theoretical meta-materials constructed from Pendry’s SRRs using the computer program. This was done for the SRRs only, and also for a LHM (Figure 8). The results of these simulations also show a stop-band near resonant frequency in the

Figure 6: Results of our simulation of Smith’s SRR experiment

Figure 7—Results of our simulation of Smith’s LHM experiment.

Figure 8—Simulation of an LHM made from Pendry’s sample SRR (same construction as in figure 1) and a conducting wire structure (plasma frequency = 20 GHz)
SRRs, and a pass-band over the same range in the LHM, as is seen in the Smith experiment. These calculations were done primarily for the purpose of understanding the transmission, reflection, and absorption curves and their dependence on certain parameters.

CONCLUSION

The computer program has been shown to work for known results and, given its success at modeling both experiments mentioned in this paper [2, 5], seems to be giving correct data in all cases. Of course, these simulations do not prove the strange effects predicted by Veselago [1], such as the reversal of the Doppler effect; more experiments need to be done in order to confirm such behavior. However, the success of the program to describe LHMs supports the case that a material exhibiting simultaneously negative $e$ and $m$ does indeed transmit electromagnetic waves as has been predicted, and also supports Pendry’s prediction of the effective permeability of a SRR lattice.

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REFERENCES

V. G. Veselago. (1964). The electrodynamics of substances with simultaneously negative values of $e$ and $m$. Soviet Physics USPEKHI. 10, 509-514.


1 Permittivity ($e$) and permeability ($m$) relate the electromagnetic field inside a material to the field outside the media. Specifically, $D = eE$ and $B = mH$, where $E$ and $D$ are the electric fields outside and inside the material, respectively, and $B$ and $H$ are the magnetic fields outside and inside the material, respectively.